Abstract

A cosmological model is proposed. In the model, vacuum energy is converted into positive mass and the mass grows exponentially with more vacuum energy conversion up to $1.758144 \times 10^{53}$ [kg] when it stops after $9.76317 \times 10^{-14}$ [s]. During this period of mass increase the quanta form a condensate Bose-Einstein.

Introduction

Inflation refers to a period of accelerated expansion of the Universe which overcomes the horizon problem and the flatness problem associated with the conventional hot big bang model. Peebles and Vilenkin [1] presented a model called quintessential inflation which includes two accelerated phases: the inflation and the quintessence. The inflation as well as the quintessence are modeled using scalar fields and the authors of [1] presented a potential field that results in a scalar field suitable for both inflation and quintessence. In Haro [2] the replacement of the original quartic piece of the inflationary potential by the quadratic one is investigated and the evolution of the model is studied up to the present. The potential found in [1] is

$$V = \begin{cases} 
\lambda (\phi^4 + M^4) & \text{for } \phi < 0 \\
\frac{\lambda}{\phi^2 + M^4} & \text{for } \phi > 0,
\end{cases}$$

and the proposed potential in [2] is

$$V_\phi(\varphi) = \begin{cases} 
m^2 (\varphi^2 - M_\mu^2 + M^2) & \text{for } \varphi \leq -M_\mu \\
\frac{m^2}{2} M^{n+2} & \text{for } \varphi \geq -M_\mu,
\end{cases}$$

where $\lambda$ and $M < M_\mu$ are two free parameters of the model in (1). See [2] for details on (2). The calculations using (2) requires (i) the redshift at the matter-radiation equality, (ii) the matter energy density to the critical one at the present time and (iii) the value of $H_0$. The parameters obtained by $\Lambda$ CDM based Planck’s estimation were used in [2].

The potential (2), more complex than (1), was developed to avoid the problems mentioned in [2]. Let us notice that the phenomenological characteristics existent in (1) are increased in (2). This suggests investigating another process to explain the expansion of the Universe.

For the standard model of cosmology ($\Lambda$ CDM) the Universe consists primarily of cold non-relativistic dark matter, and its volume expansion is currently driven by dark energy. The $\Lambda$CDM together with the inflationary theories predicts the temperature fluctuations observed in the Cosmic Microwave Background radiation (CMB) with a...
high degree of accuracy. These fluctuations correspond to the distribution of matter in the early Universe, the primordial large scale structures of the Universe left marks on the CMB, and they can be predicted as a function of the temperature fluctuations.

Nevertheless, there are observations in contrast with those expected by the model. Using $\Lambda$CDM and the CMB map to calculate the Hubble constant, values were found that disagreed with calculations based on telescope observations of supernovae and pulsating stars. These calculations of the Hubble constant with $\Lambda$CDM and the CMB map are smaller than the stellar and supernovae versions by more than 5 percent.

In [5] we found a value for Hubble constant consistent with the values found from telescope observations of supernovae and pulsating stars. In this work results of [3], [4] and [5] are used to propose a cosmological model.

The proposed model considers a hyperspace-time $(\alpha, \beta, \gamma, \sigma, \tau)$, denoted by $\mathcal{H}$, where $\alpha$, $\beta$, $\gamma$ and $\sigma$ are spacial dimensions. At a given instant $\tau$, vacuum energy is converted into positive mass in the vicinity of a point of $\mathcal{H}$. In this way a space-time $(x, y, z, t)$, denoted by $S$, is created and after that the positive mass of $S$ grows exponentially with more vacuum energy conversion up to a certain value when it stops. During this period of mass increase the quanta form a condensate Bose-Einstein.

The increase of mass

Let the Figure 1 be, where $p$ is a point of the space-time, $r$ is the distance from $p$, $\Delta r$ is the increment in the distance from $p$, $m(r)$ is the mass (luminous and dark) that exists in the distance from $p$ smaller than $r$, $\Delta m$ the increment of mass due to $\Delta r$ and $G$ is the gavitational constant. Using the transformations of space, time and force found in Ferreira [3] we obtained

$$m(r) + \Delta m + \frac{G (m(r) + \Delta m)}{c^2 (r + \Delta r)} + G m(r) + \frac{G (m(r))}{c^2 r} + 1 = \frac{m(r)}{r} \left( \frac{r}{c^2} + 1 \right)$$

(3)

The solution of (3) for $m$ has two roots

$$m_1 = \frac{\Delta m}{\Delta r}$$

(4)

and

$$m_2 = \frac{G m_1 G + c^2 (\Delta r + r)}{G (\Delta r + r)}$$

(5)

For

$$0 < \Delta r << r$$

(6)

the equation (5) becomes

![Figure 1: The increment of mass with the increment of the radius.](image-url)
Let \( k \) be a parameter such that

\[ \Delta m_2 = k m_2 \]  

therefore

\[ m = \frac{c^2 r}{2G} \]  

The \( m \) of (9) denotes the same mass that \( m_2 \) in (8). Let us assume that \( k \) is

\[ k = -2e^{-\frac{r_{\text{Planck}}}{c}} \]  

where \( a \in R \). With (10) we obtain

\[ m = \frac{c^2 r}{2G(e^{r_{\text{Planck}}/c} - 1)} \]  

for \( r \leq r_e \)

where \( r_e/c \) is the instant in which the increase of mass ended, this event is denoted by \( I_{\text{end}} \). Let us assume that the mass of the quanta for any \( r \) are constants and equal to

\[ m_{\text{quanta}} = \frac{m(r)}{n(r)} \]  

Where \( n(r) \) is the number of quanta for \( r \). For \( r = r_{\text{Planck}} \) and \( r = 1 \) we have

\[ m_{\text{quanta}} = \frac{c^2 r_{\text{Planck}}}{2G(e^r - 1)} \]  

Substituting (13) into (12) we found

\[ n(r) = \frac{2G(e^r - 1)}{c^2 r_{\text{Planck}}} m(r) \]  

or

\[ n(r) = \frac{r(e^r - 1)}{r_{\text{Planck}}(e^r - 1)} \]

**The value of \( r_e \)**

We adopt 1.380000×10^{10} years (13.800±0.024×10^9 years [6]) as the age of the universe. In Ferreira [5] we found for the present density of the universe the value

\[ \rho = 1.885957 \times 10^{-26} \text{ kg m}^{-3} \]  

since

\[ m = \rho (4/3\pi t_0 c)^3 \]  

where \( t_0 \) is the age of the universe, we have

\[ m = 1.758144 \times 10^{-5} \text{ kg} \]  

Using (11) and (18) we obtain
The value of $\alpha$

Let $m_{\text{quantum}}$ be confined in a well of spherically symmetric potential

$$V(r) = \begin{cases} 0 & \text{if } r \leq l_{\text{plank}}, \\ \infty & \text{if } r > 0, \end{cases}$$

(20)

The Eigen-energy for $l = 0$ and $n = 1$ is

$$E_{01} = \frac{\hbar^2 \pi^2}{2 m_{\text{quantum}} l_{\text{plank}}^2}$$

(21)

The values of $E_{01}$ for $r = l_{\text{plank}}$ and for $r = r_c$ are $4.345471 \times 10^9$ [J] and $3.324962 \times 10^{-51}$ [J] ($8.269756 \times 10^{-34}$ [eV]), respectively. For the time being let us notice that $E_{01}$ for $r = l_{\text{plank}}$ is equal to $m_{\text{quantum}} c^2$. The Figure 2 shows $E_{01}$ for $l_{\text{plank}} \leq r \leq 100 l_{\text{plank}}$.

For $r = 2 l_{\text{plank}}$ we have $E_{01} = 1.086368 \times 10^9$ [J] and we see that the value of $E_{01}$ decreases monotonously with the increase of $r$. Let us assume that for $r \geq 2 l_{\text{plank}}$ the mass of the quanta are

$$m_{\text{quantum}} = \frac{E_{01}}{c^2}$$

(22)

From (13) and (21) we have

$$m_{\text{quantum}} c^2 = c^4 l_{\text{plank}}^2 - \frac{\hbar^2 \pi^2}{2 m_{\text{plank}} l_{\text{plank}}^2}$$

(23)

substituting (13) into (21)

$$\frac{1}{e^2 - 1} \frac{\sqrt{2} G \hbar \pi}{l_{\text{plank}} c^2}$$

(24)

Solving (24) we find

$a = 0.2029959$

(25)

The increase of the kinetic energy of the quanta with $r_c$

Let $\sigma$ be an extra spatial dimension, so we can visualize in $(x, y, z, \sigma)$ the hypersurface, $\text{vol}_\sigma$, occupied by the mass $m$, expanding symmetrically. Consider that in the instant $r_c/c$, a geometric body in $(x, y, z, \sigma)$ intersects $\text{vol}_\sigma$ so that the intersection of the two generates a circumference with radius $2 r_c$. Now let $m$ be concentrated at the center of the circumference and, in analogy with a test mass orbiting a body of mass $m$, Figure 2: $E_{01}$ versus $r$. 

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in the space \((x, y, z)\), let’s write

\[
\Delta t' = k \frac{2\pi r_c}{\sqrt{G m(r_c)}}
\]

(26)

where \(k\) is a constant. Let’s assume that, at time \(r_c/c\), a particle \(q\) in the position \(P\) moves with speed \(c_r\) in the direction \(+d\) and that, for analysis purposes, the radius \(r_c\) remains constant with the time from \(r_c/c\) to \(r_c/c + \Delta t'\). Then after the \(\Delta t'\) time interval the particle returns to the \(P\) position arriving from the \(-d\) direction. Therefore

\[
c_r = \frac{2\pi r_c}{\Delta t'} - \frac{1}{k} \frac{2\pi G m(r_c)}{r_c}
\]

(27)

For \(r_c = r_c - d(r_c)\) we have

\[
k > \frac{1}{c} \sqrt{\frac{2\pi G m}{r_c}} = 3.553543 \times 10^{15}
\]

(28)

For \(r \leq r_c\), both the space curvature and the density, decrease with the increase of the radius \(r\). The Figure 3 shows the decrease of the density with the increase of \(r\) and the Figure 4 shows the increase of \(m/r\) with the radius (the equation (11) was used in both figures). The principal factor for the increase of \(c_r\) is the mass. The kinetic energy of the quanta increases with the increase of the number of quanta.

**The primordial Bose-Einstein condensate**

We assumed that for the time less than \(c/r_c\) the quanta have the same energy, now let us assume that the quanta are bosons. The magnitude of the quantum fluctuation of energy is

**Figure 3:** Energy density in function of the radius.

**Figure 4:** Mass in function of the radius.
\[ \Delta e = \frac{\hbar c}{\Delta r} \]  
(29)

or

\[ \Delta r = \frac{2 G (e^\varepsilon - 1) \hbar}{c^2 \, l_{\text{Planck}}} \]  
(30)

The Compton wavelength given by (30) is

\[ \Delta r = 7.27545 \times 10^{-36} \, [m] \]  
(31)

\( \Delta r \) is equal to 0.225079(2) \( l_{\text{Planck}} \) and considering the volume for \( r = r_c \), the ‘average distance between adjacent quanta’ is

\[ 2 \left( \frac{1}{n(r_c)} \right)^{1/3} r_c \]  
(32)

or

\[ 1.933280 \times 10^{-25} \, [m] \]  
(33)

We assume that the bosons form an ideal gas and the quanta have no zero rest mass.

For the Bose-Einstein condensate we have that the critical temperature is

\[ T_c = \left( \frac{n_v}{\zeta(3/2)} \right)^{2/3} \frac{\hbar^2}{2 \pi m k_B} \]  
(34)

where \( n_v \) is the density of particles, \( \zeta \) is the Riemann zeta function and \( k_B \) is the Boltzmann constant. From (13) we have

\[ m_{\text{quantum}} = 6.333061 \times 10^{-9} \, [kg] \]  
(35)

Using \( r_c \) given by (19) into (15) we found

\[ n(r_c) = 2.776136 \times 10^{61} \]  
(36)

and

\[ n_v = \frac{n(r_c)}{4/3 \pi r_c^3} = 2.643122 \times 10^{34} \, [m^{-3}] \]  
(37)

Using \( m_{\text{quantum}} \) as the mass of the particle into (34) we obtain

\[ T_c = 3.522439 \times 10^{12} \, [K] \]  
(38)

From the Figure 3 and considering the constancy of \( m_{\text{quantum}} \), we see that in the time interval \( (\frac{3}{c} \frac{l_{\text{Planck}}}{c}) \) the temperature \( T_c \) decreases with the increase of the time. The Figure 5 shows this.

\[ \text{Figure 5: Critical temperature of the Bose-Einstein distribution.} \]
The vacuum energy reduction

Let \( (\alpha, \beta, \gamma, \sigma, \tau) \) be a hyperspace-time, where \( (\alpha, \beta, \gamma, \sigma) \) are spacial dimensions and \( \tau \) is the temporal dimension. Let us assume that in a given instant \( \tau \), a particle of mass \(-2m\) changes to \(+m\) by tunnel effect. Now let us suppose that for some time interval \( \Delta \tau \) a number of quanta pass the path open by the first particle and, for these particles, the change from \(+m\) to \(-m\) requires \( -mc^2 \) of energy. Finally let us suppose that this irruption of quanta producing a space-time \((x, y, z, t)\) with matter have a mechanism in \( (\alpha, \beta, \gamma, \sigma, \tau) \) that ended this event.

The hypothesis that the first particle produces the space-time with mass \(+2m\) provides the energy \( E_{\text{q}} \) for the mass \( m \) and \( r = l_{\text{planck}} \).

For the time of Planck, \( t_{\text{planck}} \) the magnitude of the quantum fluctuation of energy is

\[
\Delta e = \frac{\hbar c}{l_{\text{planck}}} \tag{39}
\]

For a volume of radius \( l_{\text{planck}} \) and the energy given by (39) the energy density is

\[
\Delta e = \left( \frac{\hbar c}{l_{\text{planck}}} \right)^2 \frac{4}{3} \pi l_{\text{planck}}^3 = \frac{3\hbar}{4\pi l_{\text{planck}}^3} t_{\text{planck}} \tag{40}
\]

or

\[
\Delta e = 1.106150222 \times 10^{13} \text{ [J m}^{-3}] \tag{41}
\]

In Ferreira [4] the value of the vacuum energy is obtained with

\[
\Delta e = \frac{\pi^2 \hbar c}{45(l_{\text{planck}})^3} \tag{42}
\]

the Casimir result [7] for the force between perfectly conducting parallel plates was used and the result is

\[
\Delta e = 1.016124 \times 10^{13} \text{ J m}^{-3} \tag{43}
\]

Ferreira [5] shows that the value of the vacuum energy can be obtained with

\[
\Delta e = \frac{3c^2}{4\pi a^2 G^2 \hbar} \tag{44}
\]

or

\[
\Delta e = 1.1061305 \times 10^{13} \times a^{-2} \text{ [J m}^{-3}] \tag{45}
\]

In (44) the approximation \( R \propto r' \) (that is, \( R = k r' \)) was used. These results show the reduction in the vacuum energy with the time \( t \) in the space-time \((x, y, z, t)\).

A comparison with a ‘bi-dimensional’ universe

Let \( (\alpha', \beta', \gamma', \tau') \) be a space-time that differs from \( (\alpha, \beta, \gamma, \sigma, \tau) \) with \( \gamma = 0 \). In \( (\alpha', \beta', \gamma', \tau') \) there is a spherical surface, \( S \), which is expanding with the time \( \tau' \). The Figure 6 shows half of the spherical surface in two time instants \( \tau'_1 \) and \( \tau'_2 \). Let us consider that \( S \) is for \( (\alpha', \beta', \gamma', \tau') \) just as \((x, y, z, t)\) is for \( (\alpha, \beta, \gamma, \sigma, \tau) \). A finite number of points distributed uniformly in' the spherical surface correspond to the quanta in \((x, y, z, t)\). The vacuum energy density \((\text{energy} / \text{area})\) is uniform in’ \( S \). For the time being this spherical surface is the only body with positive mass in \( (\alpha', \beta', \gamma', \tau') \) and the vacuum energy density \((\text{energy} / \text{area})\) external to \( S \) is uniform and differs from the vacuum energy density in the spherical surface.
Temperature after the end of the irruption

After the event $I_{\text{end}}$, the $n(r)$ particles that exist collide with each other gaining kinetic energy and extinguishing the condensate Bose-Einstein. The temperature increases and, considering the energy per volume in the radiation of the black body equal to

$$\rho = m(r_c) c^2 \left( \frac{1}{4\pi r_c^2} \right)$$

we obtain

$$T = \left( \frac{1}{5.6703 \times 10^{-12} \text{ [J m}^{-3}\text{ K}^{-1}] \rho} \right)^{1/4} = 4.03591 \times 10^{10} \text{ [K]}$$

The use of $m(r_c)$ into (47) means that at $\frac{r_c}{c}$ there is no dark matter in $(x, y, z, t)$. The dark matter appears and increases with the time after $\frac{r_c}{c}$.

For a thermal spectrum characterized by the temperature given in (47) the energy of particles generating radiation with this spectrum can be estimated with

$$e_{\text{particle}} = \frac{3}{2} k_B T$$

where $k_B$ is the Boltzmann constant. Substituting (47) into (48) we have

$$e_{\text{particle}} = 5.216816 \times 10^9 \text{ [GeV]}$$

Let us notice that (49) is well above of the energy of the quark top ($173.1 \text{ GeV}$) and well below of $(10^{16} \text{ GeV})$.

Comments

There are two possible modes that result in the end of the irruption, $I_{\text{end}}$. In the first mode the origin of $I_{\text{end}}$ is in the hyperspace-time $(\alpha, \beta, \gamma, \sigma, \tau)$, that is, the source of the injection of mass into $(x, y, z, t)$ is exhausted or interrupted. The second mode occurs in the space-time $(x, y, z, t)$ due to quantum fluctuations. In principle a mode does not exclude the other, quantum fluctuations in $(x, y, z, t)$ can anticipate the end of the irruption.

The Figure 7 shows $\frac{m(r)}{n(r)}$ in function of the radius, this suggests that the probability of $I_{\text{end}}$ occurs as a function of $\frac{n(r)}{m(r)}$. 
The Figure 8 presents a comparison of the critical temperatures of the Bose-Einstein condensate, $T_c$, with the temperatures, $T_{bb}$, obtained with the equation (47), which consider the energy per volume in a black body. Let us notice that for $r < 1.7 \times 10^{-32} [m]$ the temperature $T_{bb}$ is less than the critical temperature.

References

2. Ferreira JC. The field equation, black holes and vacuum energy. 2018.